Foundations and Implications of Probability-Dependent Risk Attitudes

Dr. Thomas Epper

Department of Economics, University of Zurich
Chair of Economics, ETH Zurich

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Outline

- Modeling choice under risk: The "classical" way
- Lab and field evidence against expected utility
- Accommodating these findings: Rank-dependent expected utility
- Implications and applications
- Some words on rationality
- Conclusions

The "classical" way: Expected utility (EU) theory I

- Bernoulli (1738); von Neumann and Morgenstern (1947); Savage (1954)
- Posits that decision maker (DM) acts *as if* maximizing expected utility if he adheres to certain postulates (axioms) (Samuelson, 1952; Savage, 1954)
- DM chooses one alternative from a set of alternatives
- These alternatives are probability distributions over some outcomes ($\Rightarrow$ *lotteries*)

➥ Examples: Gambles, assets, projects, etc

- Decision rule: Choose alternative $L_i = (x_{i1}, p_1; ...; x_{im}, p_m)$ which maximizes

$$V(L_i) = \sum_{j=1}^{k} p_j u(x_{ij}) \, ,$$

➥ where utility u entails the DMs risk attitudes

The "classical" way: Expected utility (EU) theory II

- EUT as a normative benchmark

➥ Widely used in finance, macroeconomics, game theory, etc

- For small risks, risk premium is approximately proportional to variance of payoffs (Pratt, 1964; Arrow, 1965)

➥ Mean-variance model (risk-return tradeoff)

What is wrong with EU theory?

- Can lead to implausible predictions (Rabin, 2000)
- Is a poor descriptive theory of choice under risk (Starmer, 2000)

➥ Prediction of market outcomes (e.g. insurance), sorting (e.g. employment contracts), mitigating climate change

Experimental evidence against EU

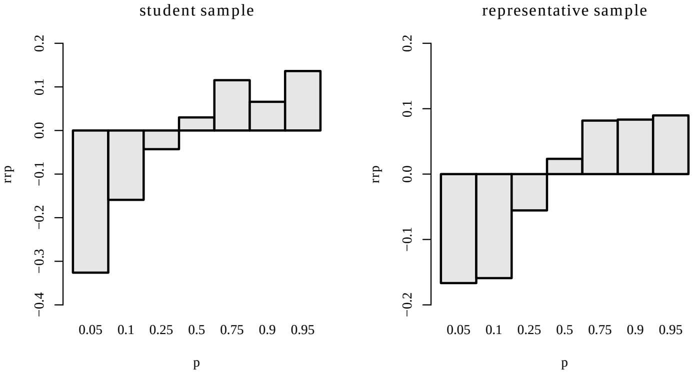
- Allais (1953) paradox (see also Kahneman and Tversky (1979))
- Which lottery do you prefer?
  - Pair 1: $A = (\$3'000, 100\%)$ or $B = (\$4'000, 80\%; \$0, 20\%)$
  - Pair 2: $A' = (\$3'000, 25\%; 0, 75\%)$ or $B' = (\$4'000, 20\%; \$0, 80\%)$
  - ➥ Majority of subjects prefer A over B, but B' over A'

- Evidence from lab and online experiments: Bruhin, Fehr-Duda, and Epper (2010), Fehr-Duda and Epper (2012), Epper and Fehr-Duda (2013b)
- Choices between simple lotteries $L = (\text{CHF}x, p\%; \text{CHF}y, 100\% - p\%)$ and a menu of certain outcomes between CHFx and CHFy

➥ Example: $L = (\text{CHF}50, 20\%; \text{CHF}0, 80\%)$ vs. CHF10

Experimental evidence

- Relative risk premium: rrp $= \frac{\mathbb{E}(L) - \mathsf{ce}(L)}{|\mathbb{E}(L)|}$



➥ Probability of better gain small $\Rightarrow$ risk seeking

➥ Probability of better gain large $\Rightarrow$ risk averse

Field evidence

- Accumulating evidence that theories allowing for nonlinearities in probabilities outperform EUT:
  - Financial markets: Kliger and Levy (2009); Polkovnichenko and Zhao (2013) option prices
  - Betting markets: Jullien and Salanié (2000); Snowberg and Wolfers (2010) horsetrack data
  - Insurance markets: Barseghyan, Molinari, O'Donoghue, and Teitelbaum (2013) consumers' deductible choices in car and home insurance policies

How to solve EUT's deficits

- Tradeoff between retaining as much mathematical tractability as possible *and* being able to better describe actual choices
- Testing the underlying postulates using choice data (Burghart, Epper, and Fehr (2013), and others). Example: monotonicity $(\$100, 20\%; \$0, 80\%) \prec (\$100, 80\%; \$0, 20\%)$

➥ Result: Only the independence axiom fails

➥ Consequence for modeling: Replace independence by a weaker assumption

The independence condition

- States that preferences between lotteries should be independent of those events for which the lotteries give the same outcome
- Independence assumption exists in EUT, but not in consumer theory

➥ Why? There are obvious complementarities when considering commodity bundles $B = (Apple, 1; Banana, 1)$, but such complementarities should not exist across in lotteries $L = (Apple, 50\%; Banana, 50\%)$ (Samuelson, 1952)

A descriptive theory of choice under risk: Rank-dependent expected utility (RDEU)

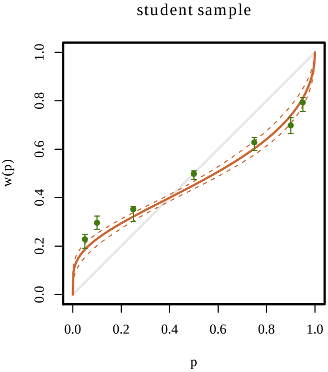
- Introduced by Quiggin (1982), building block of cumulative prospect theory Tversky and Kahneman (1992)
- Cumulative probabilities are transformed using (potentially nonlinear) probability weighting function w
- Independence is weakened: If a common outcome of two lotteries is changes into another common outcome without affecting the rank-order of the outcomes, then preferences are not affected

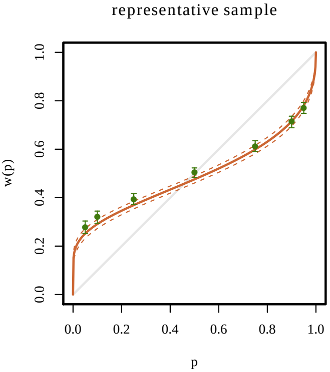
➥ Equivalent to representation where RDEU is maximized

$$V(L_i) = \sum_{j=1}^{m} \pi_j u(x_{ij})$$

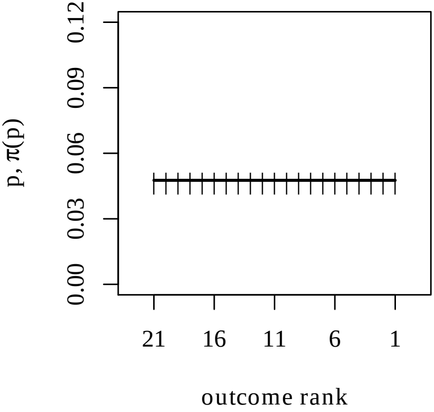
$$\pi_j = \begin{cases} w(p_1) & \text{for } i = 1 \\ w\left(\sum_{k=1}^{i} p_k\right) - w\left(\sum_{k=1}^{i-1} p_k\right) & \text{for } 2 \leq i \leq n \end{cases}$$

Probability-dependent risk preferences

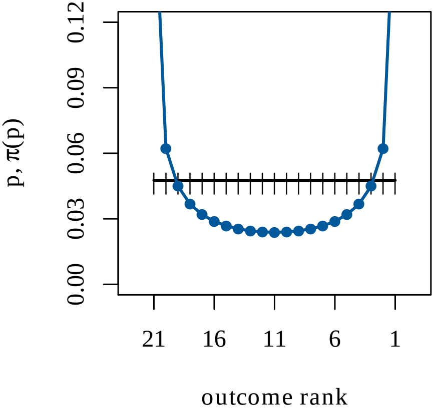




Transforming distributions



Transforming distributions



Implications I

- Theory predicts that risk premium for small risks is proportional to the standard deviation of the lottery (and *NOT* its variance (Pratt, 1964; Arrow, 1965)!)

➥ Explains ...

  - willingness to buy small-scale insurance at exorbitant prices (Segal and Spivak, 1990)
  - lesser tendency to diversify towards a risky asset while holding a safe asset (Segal and Spivak, 1990; Epstein, 1992)
  - equity premium puzzle (Epstein and Zin, 1990; Epper and Fehr-Duda, 2012)

Implications II

- Theory can predict skewness preference at any wealth level

➥ Explains ...

  - Household underdiversification puzzle (Polkovnichenko, 2005)
  - Favorite-longshot bias in betting markets (Jullien and Salanié, 2000)

- Further results:
  - Climate policy (Epper and Fehr-Duda, 2012)
  - Intertemporal risk aversion (Epper and Fehr-Duda, 2013a)

Are there rational reasons for non-EU behavior?

- Gilboa (2010): Subjective vs. objective rationality
- MacCrimmon (1968); Slovic and Tversky (1974) gave subjects the opportunity to reconsider choices that violated various axioms of EUT

➥ Subjects were not willing to change their choices violating independence (but other axioms)

- Explanations for why this can be rational: Exogenous constraints, goal-setting (Epper, 2013)

Conclusion

- There is strong lab and field evidence against EUT (independence!)
- Risk preferences depend nonlinearly on outcome probabilities
- First-order risk aversion and skewness preferences are important in understanding various real-world phenomena
- Predicting economic behavior is essential for designing appropriate policies ($\Rightarrow$ climate change mitigation) or contracts, and understanding market outcomes

A typical choice menu

	Option A	Your Choice		Option B *(guaranteed reward)*
1	Gain of CHF 50 with a probability of 75% *and* Gain of CHF 10 with a probability of 25%	A ○	● B	CHF 50
2		A ○	● B	CHF 48
3		A ○	● B	CHF 46
4		A ○	● B	CHF 44
5		A ○	● B	CHF 42
6		A ○	● B	CHF 40
7		A ○	● B	CHF 38
8		A ●	○ B	CHF 36
9		A ●	○ B	CHF 34
10		A ●	○ B	CHF 32
11		A ●	○ B	CHF 30
12		A ●	○ B	CHF 28
13		A ●	○ B	CHF 26
14		A ●	○ B	CHF 24
15		A ●	○ B	CHF 22
16		A ●	○ B	CHF 20
17		A ●	○ B	CHF 18
18		A ●	○ B	CHF 16
19		A ●	○ B	CHF 14
20		A ●	○ B	CHF 12

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